OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Wednesday **9 JUNE 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1

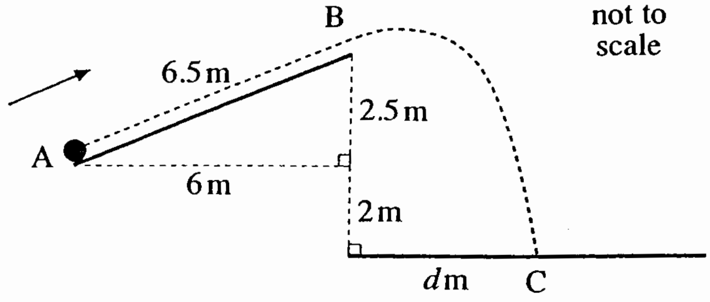


Fig. 1

A particle of mass 2 kg is sliding **up** a smooth slope of length 6.5 m from A to B, as shown in Fig. 1.

(i) By using Newton's second law, show that the acceleration of the particle is $-\frac{49}{13}\,\mathrm{m\,s^{-2}}$, where the direction AB is positive. [4]

At A, the particle has a speed of $25\,\mathrm{m\,s^{-1}}$.

(ii) Calculate the speed of the particle at B. [2]

(iii) Show that, at B, the particle has horizontal and vertical components of velocity of $\frac{288}{13}\,\mathrm{m\,s^{-1}}$ and $\frac{120}{13}\,\mathrm{m\,s^{-1}}$ respectively. [2]

The particle leaves the slope at B and becomes a projectile subject to negligible air resistance. The particle lands on horizontal ground at the point C, 4.5 m below B and d m horizontally from B, as shown in Fig. 1.

(iv) Show that the time t seconds that the particle is in the air satisfies the equation

$$637t^2 - 1200t - 585 = 0.$$ [4]

(v) Calculate the value of d. [4]

[Total 16]

2 (a) A book of mass 1.5 kg is at rest on a smooth plane at 15° to the horizontal. It is held in equilibrium by a horizontal force P, as shown in Fig. 2.1.

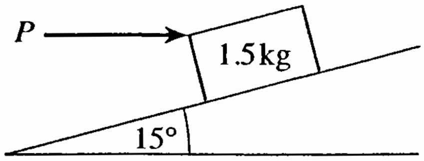


Fig. 2.1

(i) Write down an equation for the equilibrium of the book parallel to the plane. Hence calculate the value of P. [3]

(ii) Calculate the value of the normal reaction of the plane on the book. [3]

(b) **Take $g = 10\,\mathrm{m\,s^{-2}}$ in this part-question.**

A box of mass 4 kg is suspended by two light strings with tensions $\mathbf{T}_1$ and $\mathbf{T}_2$, as shown in Fig. 2.2, where the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical, respectively.

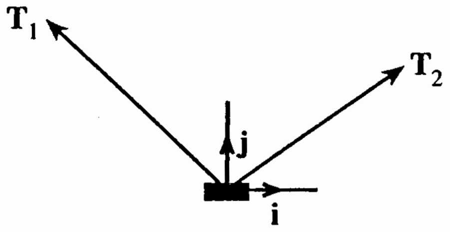


Fig. 2.2

The weight of the box, $\mathbf{W}$, is $u\mathbf{j}$ N.

(i) Write down the value of u. [1]

The box is in equilibrium.

(ii) Write down an equation connecting $\mathbf{W}$, $\mathbf{T}_1$ and $\mathbf{T}_2$. [1]

The tension $\mathbf{T}_1$ has magnitude 25 N in the direction $-3\mathbf{i} + 4\mathbf{j}$.

(iii) Show that $\mathbf{T}_1 = -15\mathbf{i} + 20\mathbf{j}$ and hence calculate the magnitude of $\mathbf{T}_2$ and the angle that $\mathbf{T}_2$ makes with the $\mathbf{i}$-direction. [6]

[Total 14]

3

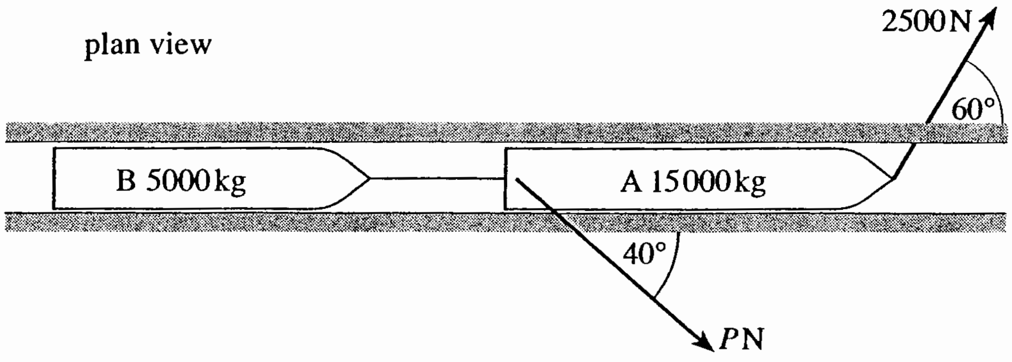


Fig. 3

A canal boat A of mass 15 000 kg is pulling a second boat B of mass 5000 kg through a straight, narrow channel. At all times the motion of the boats is parallel to this channel and the boats are connected by a light, horizontal and inextensible wire parallel to the channel.

Boat A is being pulled by two light, horizontal ropes with tensions 2500 N and P N, as shown in Fig. 3.

At one time, $P = 0$ and the acceleration of the two boats is $0.05\ \mathrm{m\,s^{-2}}$.

(i) Why must some further horizontal force act perpendicular to the channel? [1]

(ii) Show that the net forward force on the two boats together is 1000 N. [2]

(iii) Hence show that, parallel to the channel, the resistance to motion of the two boats together is 250 N. [3]

P is now increased so that the forces in the two ropes pulling the boats have their resultant parallel to the channel. The resistance to motion parallel to the channel remains 250 N.

(iv) Calculate P and the new acceleration of the boats. [5]

In a new situation in the channel, the two boats start from rest and accelerate uniformly to $2\ \mathrm{m\,s^{-1}}$ in 20 seconds. Parallel to the channel, the resistance to motion of boat A is 100 N and that of boat B is 150 N.

(v) Calculate the tension in the wire connecting the boats. [4]

[Total 15]

4 An insect moves in a straight line. The time, t, is in seconds and distance travelled is in metres.

The velocity, $v\,\mathrm{m\,s^{-1}}$, of the insect is given by

$$v = t^2 - 4t, \qquad 0 \leqslant t \leqslant 6,$$

$$v = c, \qquad 6 \leqslant t \leqslant 10,$$

$$v = at + b, \qquad 10 \leqslant t \leqslant 15.$$

You are also given that $v = 4$ when $t = 12$.

(i) Show that $c = 12$. [2]

(ii) Calculate the values of a and b and briefly describe the motion of the insect in the interval $10 \leqslant t \leqslant 15$. [4]

(iii) Calculate the values of v for $t = 0$, $t = 2$ and $t = 4$. Sketch the v-t curve for the motion of the insect in the interval $0 \leqslant t \leqslant 6$. [3]

(iv) Calculate the **distance** travelled by the insect in the interval $0 \leqslant t \leqslant 6$. [6]

[Total 15]

Mark Scheme

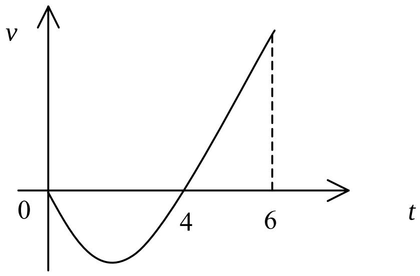
Q 1		mark		sub
(i)	$-2g\sin\theta = 2a$ or $-g\sin\theta = a$	M1	N2L parallel to slope. Accept $F = mga$ [$a = \pm g$ gets M0]	
		B1	LHS correct (allow +/-)	
	$\sin\theta = \frac{2.5}{6.5}$ or equivalent	B1	May be implied (e.g. from seeing θ = 22.6…°)	
	so $a = -9.8\times\frac{2.5}{6.5} = -\frac{49}{13}$	E1	- ve sign must be clearly established in an equation	
			See note at foot of page	4
(ii)	$v^2 = 625 - 2\times\frac{49}{13}\times 6.5$	M1	Use of appropriate *uvast* . Condone error in sign	
	so $v = 24$	A1	cao	2
(iii)			See note at foot of page.	
	$\uparrow$ $24\sin\theta = 24\times\frac{2.5}{6.5} = \frac{120}{13}$ m s^{-1}	E1	Properly established	
	$\rightarrow$ $24\cos\theta = 24\times\frac{6}{6.5} = \frac{288}{13}$ m s^{-1}	E1	Properly established	
			SC1 24 sin 22.6 and 24 cos 22.6 seen. Allow sin $\leftrightarrow$ cos and **their** ans to (ii)	2
(iv)	Vertically $y = \frac{120}{13}t - 4.9t^2$	M1	Use of appropriate *uvast* with ± 9.8 or ± 10 and $\frac{120}{13}$ or decimal equivalent	
		A1	NB no FT as u is given. Allow decimals	
	Need $-4.5 = \frac{120}{13}t - 4.9t^2$	M1	Equating **their** y to $\pm$ appropriate height	
	$\times 130$ gives $637t^2 - 1200t - 585 = 0$	E1	Clearly established using fractions	4
(v)	Solving for + ve root	M1	Some evidence required of appropriate method	
	$t = 2.28563\ldots$	A1	Award SC2 WW for this value seen (allow 2sf or better)	
	$d = \frac{288}{13}t = 50.635\ldots$	M1	Use of **their** $t\ \times$ given horiz cpt of velocity (may be given as a decimal)	
	so 50.6 (3 s. f.)	A1		4
			total	16

In parts (i) and (iii), use of non-fractional expressions must be followed in each case by a decimal equivalent to at least 3 s. f. for full credit. If fewer than 3 s. f. used, penalise this once only.

We expect a claim that the decimal is the required fraction but do not require a statement of the decimal equivalent of the required fraction.

Q 2		mark		sub
(a) (i)	$P\cos 15 = 1.5g\sin 15$	M1	Attempted resolution of at least one of the forces in an equation. Allow $\sin \leftrightarrow \cos$. No extra forces. condone g omitted.	
		B1	Either term correct. May be seen on a diagram.	
	P = 3.93885… so 3.94 N (3 s. f.)	A1	[Using force triangle or Lami: B1 setting up, B1 ans]	3
(ii)	**either** resolve vertically $R\cos 15 = 1.5g$	M1	Clearly resolving vertically. Attempt to resolve R. Allow $\sin \leftrightarrow \cos$. No extra forces. Must use weight.	
		A1	Correct	
	R = 15.2185… so 15.2 N (3 s. f.)	A1		
	or resolve perpendicular to the plane $R = P\sin 15 + 1.5g\cos 15$	M1	Both RHS terms attempted at least one with resolution. Allow $\sin \leftrightarrow \cos$. No extra forces. Condone sign error. Must use weight.	
		A1	FT **their** P only	
	= 15.2185… so 15.2 N (3 s. f.)	A1	FT **their** P only [Using force triangle or Lami: M1 A1 setting up equation, A1 ans]	3
(b) (i)	$u = -40$	B1		1
(ii)	$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = \mathbf{0}$	B1	Condone 0 instead of **0**	1
(iii)	$\left\| -3\,\mathbf{i} + 4\,\mathbf{j} \right\| = 5$	M1	Finding modulus or direction	
	so $\mathbf{T}_1 = \frac{25}{5}(-3\,\mathbf{i} + 4\,\mathbf{j}) = -15\,\mathbf{i} + 20\,\mathbf{j}$	E1	No statement about direction required [SC1 for 5(-3 **i** + 4 **j**) seen without explanation]	
	$\mathbf{T}_2 = -\mathbf{W} - \mathbf{T}_1 = 15\,\mathbf{i} + 20\,\mathbf{j}$	B1	Award if correct components seen. FT (i) and (ii).	
	$\left\| \mathbf{T}_2 \right\| = \sqrt{15^2 + 20^2} = 25$	B1	Award for modulus of **their** $\mathbf{T}_2$	
	direction is $\arctan\frac{4}{3}$ = 53.1° (3 .s. f.) with **i**	M1	Use of arctan. Award for use on **their** $\mathbf{T}_2$	
		F1	FT. Must give direction with + **i**. 3rd and 4th quadrant values may be given as +ve	6
			total	14

Q 3		mark		sub
(i)	As 2500 N force has component perpendicular to the bank the boat(s) would have component of acceleration in this direction.	E1	Some reference to no acceleration perpendicular to the bank or there is a component of the 2500 N. Accept *the boats don't hit the bank* or *the acceleration is parallel to the channel*	1
(ii)	$F = 20000 \times 0.05$ $= 1000$	M1 E1	Use of N2L with 20000 or 20000g. No extra forces.	2
(iii)	$2500\cos 60 - R = 1000$ $R = 250$ so 250 N	M1 B1 E1	Equating force to 1000 N. All forces present Condone no resolution and sign errors and $\sin \leftrightarrow \cos$. No extra forces. For the component $2500\cos 60$ [SC2 for $1250 - R = 1000 \Rightarrow R = 250$]	3
(iv)	$P\sin 40 = 2500\sin 60$ $P = 3368.24\ldots$ so 3370 N (3 s. f.) $1250 + P\cos 40 - 250 = 20000a$ $a = 0.17901\ldots$ so 0.179 m s^{-2} (3 s. f.)	M1 A1 M1 F1 A1	Equilibrium equation perpendicular to bank with an attempt at resolution of both forces. Allow $\sin \leftrightarrow \cos$. No extra forces. Accept 2 s.f. or better N2L, $F = ma$. Both rope terms attempted with resolution. Allow $\sin \leftrightarrow \cos$. Condone sign errors and resistance omitted. No extra forces. All correct. FT **their** *P*. cao. Accept 2 s.f. or better.	5
(v)	$a = \frac{2}{20} = 0.1$ so 0.1 m s^{-2} For B $T - 150 = 5000 \times 0.1$ $T = 650$ so 650 N	M1 A1 M1 A1	Attempt at *a* from the information N2L on one boat. All terms present. Condone sign errors. Accept $a = 2$ but not 0.05. Accept 2 equations which give *T*. No extra forces. FT **their** *a* (not 2)	4
			total	15

Q 4		mark		sub
(i)	$c = 6^2 - 4 \times 6 = 12$	M1 E1	Attempt at $v(6)$	2
(ii)	$4 = 12a + b$ $12 = 10a + b$ Solving $a = -4$ and b = 52 Constant acceleration (of -4 m s^{-2})	M1 M1 A1 E1	Obtaining at least one equation Solving Both correct Accept *constant accn* (ignore value), *deceleration*, *changes direction*	4
(iii)	0, -4, 0 	B1 B1 B1	All of them (may be implied from graph) General shape correct (parabola and part below t axis) t intercepts correct and graph extends to $t = 6$	3
(iv)	Need $(-)\int_0^4 + \int_4^6$ $= -\left[\frac{t^3}{3} - 2t^2\right]_0^4 + \left[\frac{t^3}{3} - 2t^2\right]_4^6$ $= -\left(\frac{64}{3} - 32\right) - 0 + \left(\frac{216}{3} - 72\right) - \left(\frac{64}{3} - 32\right)$ $= 21\frac{1}{3}$ m	M1 M1 A1 B1 A1 A1	Recognise need to divide domain Integration of v used. Neglect limits. Allow numerical method Neglect limits Limits used and correct for both parts Correct substitution of their limits into at least one integral or at least one arbitrary constant found (arb const of zero need not be established and substitution of zero limit need not be shown). cao [Award SC3 if seen WW]. Both domains used. Award SC3 for correct **displacement** of zero **provided** correct working is seen. [Numerical methods may be awarded M marks. Award other marks only if working is to 3 s. f. or better]	6
				15

Examiner’s Report

2607 Mechanics 1

General Comments

This paper was thought to be too hard by many centres and the grade thresholds were determined at the award with this in mind.

Both Q1 and Q2(a) presented a far greater challenge than intended. In both cases it was thought that the situations being examined were standard and so would be familiar to candidates and not provide too great an entry threshold to the question. This turned out not to be the case; although many candidates did navigate their way through them with ease, many more struggled to get started in Q1(i), partly recovered later in that question and then struggled again in Q2(a). On top of this, the use of fractions in Q1, which was intended to be helpful, seems to have been taken as symbolic of the question being hard.

Of course, many candidates did well throughout the paper and there were good answers seen to every question.

Quite common weaknesses seen were the inability to work with vectors in Q2(b) and the use of inappropriately approximate methods in Q4(iv). Some candidates presented their work so poorly they handicapped themselves. Many candidates struggled with all the basic arithmetic, solution of equations and trigonometry and quite a few seemed to have little knowledge of the content of the unit. It was pleasing to see fewer wrong results being used (e.g. $s = u + at$, $v^2 = u + 2as$ etc).

Comments on Individual Questions

Q.1 It has already been mentioned that the fractions caused many problems. It was acceptable to use decimal equivalents as long as sufficient accuracy was maintained.

In part (i), many of the candidates could make little progress, even to the point of not realizing that the particle was decelerating because of its weight; these often just found the angle of the slope in degrees (for some credit). Those who got started often did reasonably well; they accurately found the component of weight along the slope and the most common error was failure to establish the negative sign properly. It was interesting to see quite a few candidates using the work-energy principle (which is not in the syllabus) to derive the result; many of these were also unable to give a convincing reason for the negative sign.

Part (ii) was usually done well with the most common error being the use of *g* as the acceleration up the slope.

Part (iii) was also answered well by many candidates but quite a few lost one mark because of premature approximation when trying to establish decimal equivalence to the given fractions.

Many candidates knew how to proceed in part (iv) but a common error was to get the sign of the displacement wrong. Some candidates used the speed or the acceleration of the particle along the slope instead of the *y* component and *g*, respectively.

In part (v), a surprising number of candidates were unable to solve the given quadratic equation. Those candidates who could and who used the given equation normally obtained the correct time and value of *d*. Some candidates thought there was a horizontal component of acceleration.

Q.2 Part (a)(i) was done quite well by those who got started but a large number failed to resolve one of the forces (usually *P*) and quite a few exchanged sine and cosine.

In part (a)(ii), most of the candidates who attempted the part chose to resolve perpendicular to the plane instead of vertically (which is simpler). There were few correct answers as most of the candidates omitted one of the forces or made sign errors.

In part (b), many candidates were unable to work with vectors expressed in the form given and saw no distinction between, say, $\mathbf{T}_2$ and T_2.

In part (b)(i) the usual answer given was $u = 40$ instead of $u = -40$.

In part (b)(ii) most candidates gave $\mathbf{T}_1 + \mathbf{T}_2 = \mathbf{W}$ instead of $\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = \mathbf{0}$, presumably being confused by the tensions acting upwards and the weight acting downwards.

There was a lot of confusion seen in part (b)(iii). In order to find $\mathbf{T}_1$, some candidates chose to 'scale up' the given vector but didn't say why they were multiplying by 5. Many candidates instead found the angle of the direction vector and used the fact that the magnitude was 25 but then often didn't fully establish why the **i** component was negative.

In finding $\mathbf{T}_2$ many candidates were able to follow through from their incorrect (i) and (ii). Those who obtained a vector for $\mathbf{T}_2$ usually went on to find the correct magnitude and direction of their force. A large number of candidates forgot to find the magnitude (perhaps this was just oversight). Many of the weaker candidates lost the vector form and thought that $\mathbf{T}_2$ was the difference between the magnitudes of **W** and $\mathbf{T}_1$ or was a scalar obtained from the components.

Q.3 The answers to part (i) were usually awarded a mark but some candidates just gave an argument demonstrating that a resistance must be present. Given diagrams such as in this question, many candidates fail to distinguish between parallel, perpendicular, horizontal and vertical. Despite the diagram being labelled 'plan view', many candidates treated the direction up the page as being vertical; presumably they thought that one of the rope handlers was levitating and the other was a mole!

Part (ii) was answered correctly by most of the candidates.

Part (iii) was answered correctly by most of the candidates but some obtained a negative answer and failed to explain the sign.

Finding *P* in part (iii) was poorly done by many of the candidates. Some of them attempted to form two equations of motion (both horizontal, one along and the other normal to the canal) using a common acceleration. However, the most common error was to exchange sine and cosine and to obtain $Pcos40 = 2500\cos 60$. In finding the acceleration, some candidates forgot to resolve their *P* or forgot to include the resistance. Many candidates were able to write the correct equation of motion but could not make any further progress as they didn't know how to find *P*. It was pleasing that, despite these errors many candidates obtained full marks.

In part (iv), almost all candidates who didn't (incorrectly) use *a* = 2 found the correct acceleration. Those who used Newton's second law on boat B had few problems in finding the tension. However, those who used boat A had also to find the forward force and made mistakes when trying. A common error was to forget the resistances and to write it as 20000×0.1.

Q.4 Most of the candidates saw that the motion was 'joined up' and so found *c* in part (i).

Part (ii) was also generally done quite well but some candidates wrote only one equation and simply rearranged and substituted into the original form and kept manipulating until they made a mistake and then found values for *a* and *b*. A number of candidates forgot to comment, as requested, on the motion of the insect; of those that did, many commented correctly on the *constant acceler*ation or *deceleration* or *change in direction* but some falsely said that the insect *slowed down* (it does so initially and then speeds up again).

Almost all of the candidates found the correct values for *v* in part (iii) but many of the sketches were poor. Some candidates only considered the interval from *t* = 0 to *t* = 4 and many thought that the points should be joined with straight line segments (this was not an error restricted to otherwise weaker candidates).

The solutions to part (iv) were normally of three types: those who integrated appropriately, those who integrated to obtain the displacement instead of the distance and those who used numerical methods. Those who integrated usually did so accurately and the number of marks obtained depended on whether they realized the need to split the domain (about half of them). Credit was given for the use of numerical methods but few candidates worked to sufficient accuracy to score more than two marks; it was common to see the domain divided into just two triangles.